Acknowledgments

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Improved Pilot Model for Space Shuttle Rendezvous

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Introduction

RENDEZVOUS with the space station will present some unique problems. Of particular concern is the final portion of the mission, when the Shuttle is within approximately 125 m of the station. During this phase of rendezvous the Shuttle's jet plume could easily damage the station's large solar panels.

Unfortunately, it is difficult to analyze the effects of the Shuttle's jet plume with existing methods because a human pilot makes all targeting decisions during the final portion of rendezvous. NASA is conducting numerous human-in-the-loop simulations, but they are very time consuming and cannot generate sufficient data.[1] In addition, due to the variability associated with any human operator, parametric studies using human-in-the-loop simulations require very large Monte Carlo-type analysis. This short-term problem could become much worse if preliminary analyses indicate new piloting procedures are necessary for space station missions. If so, many additional simulations will be required to analyze each proposed change.

This Note presents a software pilot model as a solution to these problems. Its performance is compared with a number of human-in-the-loop simulations and with a similar pilot model based on Boolean logic. This evaluation demonstrates that the fuzzy logic pilot is a good model of a human pilot and is capable of supplementing NASA's database. Because this pilot model provides exact repeatability, it would also be beneficial for parametric studies used to evaluate proposed piloting rules or techniques. During these simulations, a few critical parameters could be varied, holding everything else constant. This is not possible with human-in-the-loop simulations due to the variability associated with human pilots.

Development of the Pilot Model

Unlike other fuzzy logic pilot models, which are intended to be superior to a Shuttle pilot and use navigational data that are not available to astronauts, the model presented in this Note is designed to emulate an astronaut's performance.[2–5] This model, therefore, only uses the sensory data available to an astronaut.

A pilot has three references of relative position and velocity during the terminal phase of rendezvous. The best reference is simply the view out the window. To assist the pilot, cross hairs, referred to as the crew optical alignment sight (COAS), are mounted in the window. The view through the COAS helps the pilot keep the Shuttle within a prescribed approach corridor, typically an 8–10 deg cone extending in front of the target vehicle. A pilot determines whether vertical (or horizontal) burns are required by referencing the target's vertical (or horizontal) position and velocity in the COAS. Astronauts can also reference an instrument that displays the Shuttle's attitude rates. This information helps the crew determine whether the target's motion through the COAS is due to the Shuttle translating or rotating. The third instrument, which will be added for rendezvous missions to the space station, is a laser. It will provide range and range rate information to the target. Software models of all three of these references were used by the pilot model.

The pilot model processes these data using fuzzy logic (see Refs. 6–9 for a discussion of fuzzy logic). This logic was developed based on pilot comments, piloting rules and techniques, and simple orbital analysis. However, the primary source of information used to determine the specific fuzzy rules and set boundaries came from analyses of over 90 NASA human-in-the-loop simulations.

The fuzzy rules used by the pilot model are very simple. As with real pilots, each axis is controlled independently. The two axes used to keep the Shuttle inside the approach corridor have roughly 14 fuzzy rules each. These rules consider the estimated relative position and velocity of the Shuttle with respect to the station as seen through the COAS. A typical fuzzy rule is as follows. If the Shuttle is high and is moving up rapidly, then make two burns vertically. The terms high and up rapidly are defined by fuzzy sets. Because human pilots fly a smaller approach corridor as the distance to the target decreases, the definitions for these set boundaries differ slightly for large and small ranges, which are also fuzzy sets. In addition, another fuzzy rule inhibits the actions if the Shuttle's attitude rates are relatively high (another fuzzy set). This mimics real pilots, who know the view through the COAS is unreliable if the Shuttle has a high attitude rate. Therefore, they wait to assess the need for any burns until the Shuttle's attitude motion is relatively low.

The logic for controlling the Shuttle's closure rate uses only two simple rules. If the closure rate is fast, a single burn is made directly at the station to decrease the closure rate. Likewise, if the closure rate is slow, one burn is commanded to increase it. The set boundaries for the fuzzy terms fast and slow vary with range and correspond to NASA's piloting rules and observed pilot performance.

Results and Discussion

The first objective for any pilot model is to duplicate the performance of an average human pilot. This capability is demonstrated by comparing one simulation by the fuzzy pilot with four human-in-the-loop runs using the same initial conditions.

Figure 1a compares the trajectories in the orbital plane for these different simulations. This is a profile view of the Shuttle's approach, beginning in the lower left-hand corner and ending on the right. It uses a local vertical, local horizontal (LVLH) coordinate frame. The origin is the space station's center of mass. The X axis, shown along the horizontal scale, measures the position of the Shuttle's center of

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mass in front of the station, and the Z axis, shown along the vertical scale, measures the Shuttle's position below the station. Notice that all of the simulations began with one large hop about 100 m in front of the station. Then, through a series of smaller hops, the Shuttle approached the station, staying below the station's center of mass and within the prescribed corridor. After completing the rendezvous the Shuttle's center of mass was approximately 28 m below and 15 m in front of the station's center of mass. Notice that the solid line, which depicts the fuzzy pilot's trajectory, is average. It is a smooth trajectory that avoids the extremes of the four human-in-the-loop runs.

As a comparison, Fig. 1b shows the same four human-in-the-loop runs compared with another pilot model that uses similar rules, only these rules are based on Boolean logic. This pilot model, which took just as long to develop as the fuzzy pilot model, is not representative of the human-in-the-loop results.[1] It does not travel as high on the first hop and never drops as low as the other runs. It also has a much more abrupt control response than the humans. This is a result of the limitations associated with traditional, Boolean logic. It cannot easily combine the conclusions from a number of rules. In addition, the transition from one set to another is well defined. As a result, the Boolean pilot has an abrupt transition between its decision to not make a burn and its decision to make a burn.

As shown in Table 1, the fuzzy pilot also more closely matches the average human-in-the-loop simulation in three important categories: the total simulation time, fuel used, and total jet on-time commanded by the pilot. A comparison of 41 other parameters and 7 other trajectory plots also demonstrated that the fuzzy pilot is a better model of a human's performance.[1]

The fuzzy pilot model can also easily emulate different piloting techniques using the same decision logic. Just as different human pilots have different definitions for terms such as high, low, etc., the pilot model can also redefine these fuzzy terms. To demonstrate this, the fuzzy sets were adjusted slightly, allowing the model to mimic the highest and lowest trajectories from the four human-in-the-loop runs.[1] This capability allows the fuzzy pilot model to emulate a range of potential pilots, not just one average pilot.

The final evaluation of the fuzzy pilot compared its performance with the results from one human pilot using a number of different initial conditions. For this comparison, the fuzzy pilot flew all eight

Table 1 Comparison of two pilot models

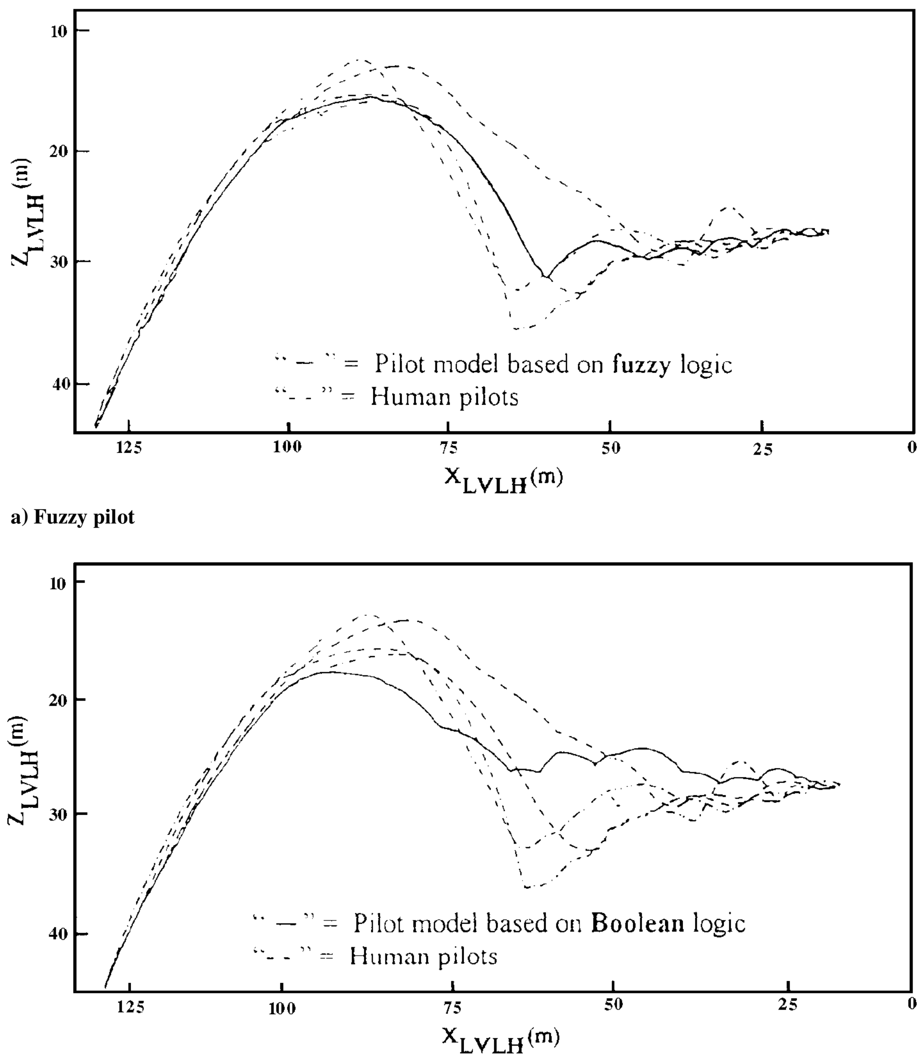
	Average human simulation	Fuzzy pilot	Boolean pilot
Time, s	1975	1966	1830
Fuel, kg	112	111	106
Jet time, s[a]	66.5	65.0	57.9

[a]Total jet on-time commanded by the pilot.

Table 2 Evaluation of the fuzzy pilot

	Human pilot		Fuzzy pilot	
	Average	σ	Average	σ
Time, s	2106	261	2139	227
Fuel, kg	101	23	104	20
Jet time, s[a]	60	14	59	12

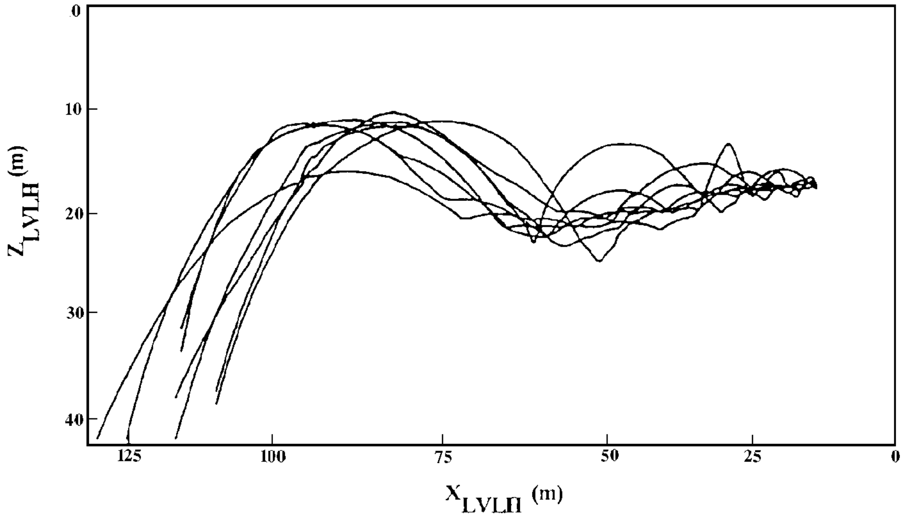
[a]Total jet on-time commanded by the pilot.



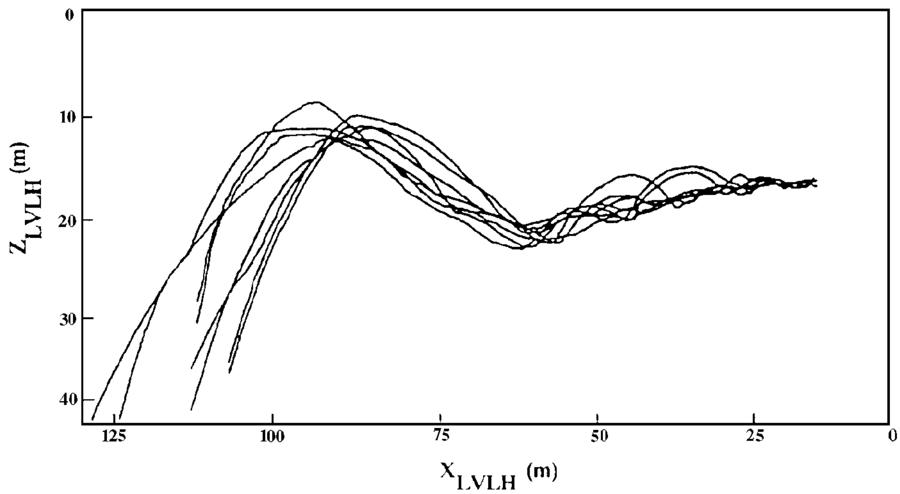
a) Fuzzy pilot

b) Boolean pilot

Fig. 1 Trajectories from two pilot models compared with four human-in-the-loop trajectories.



a) Eight simulations by a human pilot



b) Eight simulations by the fuzzy pilot

Fig. 2 Comparison of trajectories by a human pilot and the fuzzy pilot.

simulations that were flown by one human to a specific assembly stage of the space station. For this test, however, the same set of pilot parameters used by the fuzzy pilot discussed earlier could not be used because this new set of simulations was created using a different station assembly stage and a smaller approach corridor specified by newer NASA piloting requirements. Therefore, one of the human-in-the-loop simulations was used to determine the new fuzzy set boundaries. These new sets were then used for all eight simulations by the fuzzy pilot model.

The trajectory plots comparing these two sets of eight runs in the orbital plane are shown in Fig. 2. For all eight simulations the fuzzy pilot responded well and displayed typical human characteristics, without requiring modifications for each initial condition. However, the human pilot's trajectories had slightly more variation than those flown by the pilot model. This was expected because the fuzzy pilot's parameters were held constant for all eight simulations. Some variation of the fuzzy set boundaries would be necessary to accurately model the inconsistencies of a human pilot over a number of different simulations.

For this evaluation, however, the fuzzy sets were not varied, and the fuzzy pilot was very consistent. This consistency is not possible with a human pilot and is one reason the fuzzy pilot is well suited for parametric studies. Using this model, studies can be performed while varying only the parameter in question and keeping all others constant. This is not possible with human-in-the-loop simulations due to the variability associated with any human operator.

Other comparisons of these simulations were made, including the location and frequency of burns, other trajectory plots, COAS angles, closure rate, and final miss distance. All of these comparisons showed the fuzzy pilot had similar performance when compared with the human pilot.[1] As a sample, Table 2 shows the average simulation time, fuel consumed, and total jet on-time commanded by the pilot for both sets of runs. For each parameter the simulations by the fuzzy pilot are representative of the human's results.

Conclusions

A fuzzy logic control system can accurately model a human pilot during the terminal phase of Space Shuttle rendezvous. Such a model is needed to generate numerous simulations for analyses of the Shuttle's jet plume effects on the space station. The fuzzy pilot proved to be a much better model of a human than a similar model based on Boolean logic. The fuzzy pilot model displayed average performance when compared with four human-in-the-loop simulations from the same initial conditions. The fuzzy pilot also demonstrated its ability to emulate different piloting techniques by simply adjusting the definitions of a few fuzzy sets.

Finally, the pilot model demonstrated that it does not have to be adjusted for each initial condition. Using the same parameters, it produced human-like results from a number of initial conditions. This capability gives the model a great deal of consistency, which will be beneficial for future parametric studies.

Acknowledgments

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Minimum-Time Continuous-Thrust Orbit Transfers Using the Kustaanheimo–Stiefel Transformation

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Introduction

In this Note, we develop the minimum-time, continuous-thrust, planar transfer from one circular orbit to another using the Kustaanheimo–Stiefel (KS) transformation. The development is closely related to other work on minimum-time, continuous-thrust transfers[1] and makes use of the approximate initial values of Lagrange costates developed in Ref. 1. The following development is intended to provide insight into the numerical solutions.

The KS transformation[2] is intended to regularize the equations of motion in the problem of two bodies.[3] When this transformation is used in conjunction with a change of independent variable, the equations of motion in two dimensions have the form of a harmonic oscillator.[2] This allows for simple analytical solutions, which may be perturbed by other forces such as a third body or a propulsion system. If the spacecraft is propelled by continuous thrust, there are no analytical solutions to the problem. However, a perturbation approach using a small thrust parameter may provide a reasonable approximation to the exact case. To determine the accuracy of such an approximation, an exact numerical case must be available. Because the exact solution requires solving a two-point boundary-value problem, it is of interest to develop a reliable means of obtaining these solutions.

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Euler–Lagrange theory is applied to the transformed equations of motion to obtain the optimal control formulation. Based on the results of many numerical cases, the optimal initial costates are plotted against each other for a large range of initial thrust acceleration values. A specific example is provided of an Earth-to-Mars transfer, with the resulting converged values of the initial costates and the fictitious time.

Equations of Motion

We begin by developing the regularized equations of motion, including the terms due to continuous thrust acceleration $a(t) = T/(m_0 + \dot{m}t)$, where T is the constant thrust magnitude, m_0 is the initial mass, and $\dot{m}$ is the constant rate at which mass is expelled by the thruster. All quantities are expressed in canonical units,[4] where the gravitational constant μ is unity regardless of the system under consideration as long as the initial circular radius is defined to be one distance unit (DU) and the initial circular velocity is one distance unit per time unit (DU/TU). The initial acceleration due to thrust is $A = a(t_0)$, and the final desired orbit radius is $R = r(t_f)$. The equations of motion for a two-dimensional orbit are as follows:

$$\ddot{x} = -(\mu/r^3)x \tag{1}$$

$$\ddot{y} = -(\mu/r^3)y \tag{2}$$

as seen in Fig. 1.

Using the KS transformation for two dimensions, the coordinates x and y are replaced by u_1 and u_2 through the following relationship:

$$(u_1 + iu_2)^2 = x + iy \tag{3}$$

The independent variable t is replaced by the fictitious time s with the following differential equation:

$$\frac{\mathrm{d}t}{\mathrm{d}s} = r \tag{4}$$

These transformations lead to the regularized equations of motion[2]

$$u_1'' = \left[\frac{2(\boldsymbol{u}'^T\boldsymbol{u}') - \mu}{2r}\right]u_1 \tag{5}$$

$$u_2'' = \left[\frac{2(\boldsymbol{u}'^T\boldsymbol{u}') - \mu}{2r}\right]u_2 \tag{6}$$

The primes indicate differentiation with respect to s, $\boldsymbol{u} = (u_1, u_2)^T$, and $r = u_1^2 + u_2^2$. The symbol $(\boldsymbol{u}'^T\boldsymbol{u}')$ indicates an inner product. The purpose of the regularization under the KS transformation is to reduce numerical integration difficulties when r is small by placing the inverse of r into a term that represents the constant angular momentum magnitude of a two-body orbit. This term premultiplies the state variables u_1 and u_2 in Eqs. (5) and (6) but it will not remain

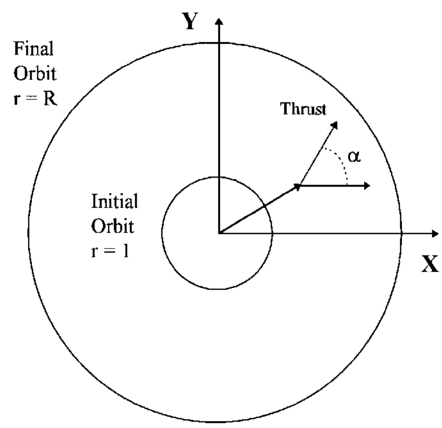

Fig. 1 Problem geometry in two dimensions.